





Conditional and Unconditional Distribution of Asset Returns with Special Reference to Skewness and Risk Management

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Foreseeable Future...? Politics/Finance/Economy.. CRISIS? • Estimation & Forecasting the Future... Not so Easy? • Seeing the Future... Horoscopes? • <u>Managing the Future</u>... Perfect if you can! • Shaping the Future... **Ultimate point? Maybe not!**

- **Student:** I notice that people sometimes use the words statistics and probability when talking about the same things. Are these two words just different names for the same concept?
- Mentor: What do you think?
- Student: I want to check a dictionary first and see what it says.
- Mentor: Check several dictionaries and based on what you find, make a definition for each word. A scientific or mathematical dictionary will give you more detailed information.

Probability: 1: being probable **2:** something that is probable **3:** a ratio expressing the chances that a certain event will occur **4:** a branch of mathematics studying chances of random events.

Statistics: 1: facts or data assembled and classified so as to present significant information **2:** collection, calculation, description, manipulation, and interpretation of the mathematical attributes of large sets or populations **3:** a branch of mathematics dealing with collection, analysis and interpretation of data.

Student: So statistics is all about data, and probability is all about chance.

Mentor: Exactly. Let me talk about probability as the measure of chance. Specialists look at this meaning of probability in two different ways that are called <u>Frequency View</u> and <u>Personal View</u> (or Subjective View, as philosophers call it).

WORKS REFERRED IN THE PRESENTATION



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Distribution Theory

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Selected Books

Addressing Skewness

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Skewness is defined to

-**describe asymmetry** from the normal distribution in a set of statistical data.
- Most sets of data, including stock prices and asset returns, have either **positive or negative skew** rather than following normal distribution (which has a skewness of zero).
- Skewness is **extremely important** to finance and investing.
- By knowing which way data is skewed, one can **better estimate** whether a given (or future) data point will be more or less than the mean.
- Most advanced economic analysis models study data for skewness and **incorporate it into their calculations**.
- **Skewness risk** is the risk that a model assumes a normal distribution of data when in fact data is skewed to the left or right of the mean.

Context and Motivation

- The calculation of market risk based capital requiremets has found a vast area of implementation in the financial markets and has become a focus of academic interest.
- The basic idea behind this new regulation, in addition to investor/depositor protection, is to minimise systemic risk through more transparency and a more sound approach to risk measurement and management.
- Consequently, financial institutions are now expected to quantify and manage financial risk in a more realistic and accurate way.
- The factors behind the new dynamic market structure are complex.
- Traditional risk management and measurement tools became insufficient to deal with the risks inherent in complex portfolios that are composed of many instruments displaying both linear and non-linear characteristics.

- Context and Motivation (cont'd)
 - One of the most important events in risk management has been the emergence and implementation of an exclusively designed category of risk measurement systems.
 - These systems include many theoretical and methodological elements.
 - Most are based on strong statistical assumptions.
 - Since there is no unique system or method that is capable of measuring risk adequately, many of these systems are employed to complement each other.
 - Each method displays different characteristics depending on the structure of the institution and the composition of the portfolio.
 - Today Value-at-Risk (VaR) has become the most popular of these systems which are used to measure the market or other portfolio risks.

Purpose

- The study of emerging markets is of interest for a number of reasons.
- Firstly, it provides an opportunity for testing the robustness of well-established empirical regularities that have been found in other developed markets.
- Secondly, the characteristics of emerging markets are often found to be very different from those of developed markets (see, for instance, Bekaert *et al.*, 1998).
- In particular, the perceived risk of emerging equity markets is much higher than that of developed equity markets, particularly for foreign investors.

Purpose

- As emerging equity markets start to comprise a larger share of world investment portfolios; the study of financial risk management in these markets becomes of paramount importance (carry trade?).
- The studies referred in this presentation are concerned with the implementation of VaR in both Turkey which is one of Europe's largest emerging markets and the US and UK to make a comprehensive comparison between a developing market and two developed markets.

Contributions of the Studies

- The similarities and differences in the conditional and unconditional distributional characteristics of Turkey and developed markets are investigated.
- Two new families of distributions the skewed generalised-t (SGT) and the exponential generalised beta (EGB) - are evaluated.
- These distributions, together with a wide range of distributions they nest are estimated in order to present a broad picture of the characteristics of the unconditional distribution characteristics of returns in both markets.
- GARCH-SGT model introduced to model the conditional distribution of equity returns.
- The results provide new evidence about the (un)conditional characteristics of both markets.



• Contributions & Findings of the Studies (cont'd)

- Recently, the issue of non-linearity and chaotic behaviour in financial asset returns has received much attention.
- The implication of non-linearity on financial risk management is a quite a new topic.
- Another contribution of these studies (particularly 2001b) has been to analyse the consequences of non-linearity in general for financial risk management.
- Exploiting both linear and non-linear dependence in asset returns should reduce the cost of implementing VaR, in the sense that the average capital required to cover against unexpected losses should be lower and more realistic).
- It is shown that by exploiting the non-linear dependence (through BDS Test) in equity returns, the cost of implementing VaR is very substantially reduced (is it good?).



Applied Financial Economics, 2004, 14, 195-202

Skewness in the conditional distribution of daily equity returns

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Context and Motivation

- Asset returns (conditional/unconditional) are important for a number of applications in finance, including risk management, asset pricing and option valuation
- In GARCH framework, it is generally assumed that returns are drawn from a symmetric conditional distribution such as normal, student-*t* or GED.
- The use of a symmetric distribution is inappropriate if the true conditional distribution of returns skewed.
- This study models the conditional distribution of daily returns in 5 int'l equity market indices and a world equity using the skewed generalised-t (SGT) distribution.
- The SGT distribution has been introduced by Theodossiou (1998) and nests three most commonly used distributions as special cases.

- Context and Motivation (cont'd)
 - The correct specification of the conditional distribution of returns is important for a number of reasons:
 - **Misspecification** of conditional distribution leads to estimates that are **inefficient** (Bollerslev, 1986)
 - Engle and Gonzalez-Rivera (1991) show that the inefficiency of QML may be substantial when the true distribution is skewed.
 - Effective risk management critically depends on true distribution of portfolio returns (value-at-risk)
 - The correct specification of the conditional distribution of asset returns is also important for asset pricing and for the valuation of contingent securities such as options.

- Skewed Generalised-t Distribution (SGT)
 - Introduced by Theodossiou (1998)
 - A flexible distribution that allows for very diverse levels of skewness
 - Used to model the unconditional distribution of daily returns for a variety of financial assets
 - SGT distribution nests, *inter alia*, the normal, Studentt and power exponential distributions that are typically used with GARCH models
 - Hence it is straightforward to test the restrictions on the SGT that these distributions imply.



The skewed generalized-*t* (SGT) distribution, introduced by Theodossiou (1998), is a skewed extension of the generalized-*t* distribution, originally proposed by McDonald and Newey (1988). The probability density function of the SGT distribution is given by

$$f(x|k, n, \lambda, \sigma^2) = \begin{cases} f_1 & \text{for } x < 0\\ f_2 & \text{for } x \ge 0 \end{cases} \text{ where}$$

$$f_1 = C(1 + (k/(n-2))\theta^{-k}(1-\lambda)^{-k} |x/\sigma|^k)^{-(n+1)/k}$$
$$f_2 = C(1 + (k/(n-2))\theta^{-k}(1+\lambda)^{-k} |x/\sigma|^k)^{-(n+1)/k}$$

and C and θ are given by

$$C = \frac{1}{2\sigma} k B\left(\frac{1}{k}, \frac{n}{k}\right)^{-3/2} B\left(\frac{3}{k}, \frac{n-2}{k}\right)^{1/2} S$$
$$\theta = \frac{1}{S} \left(\frac{k}{n-2}\right)^{1/k} B\left(\frac{1}{k}, \frac{n}{k}\right) B\left(\frac{3}{k}, \frac{n-2}{k}\right)^{-1/2}$$

with S given by

$$S = \left[1 + 3\lambda^2 - 4\lambda^2 B\left(\frac{2}{k}, \frac{n-1}{k}\right)^2 B\left(\frac{1}{k}, \frac{n}{k}\right)^{-1} B\left(\frac{3}{k}, \frac{n-2}{k}\right)^{-1}\right]^{1/2}$$

where B(.) is the beta function, $-1 < \lambda < 1$, n > 0, $\sigma > 0$ and k > 2.

The parameter σ is a scale parameter, while the parameters k and n determine the height and tails of the density, and consequently its kurtosis. The parameter λ determines the skewness, with a symmetric distribution obtaining when $\lambda = 0$. By restricting the parameters of the SGT, many other well known distributions are obtained, including those that are typically used for the conditional GARCH distribution. When k=2 and $\lambda=0$ the SGT reduces to the Student-*t*; when $\lambda = 0$ and $n \to \infty$ it reduces to the power exponential or generalized error distribution; and when k = 2, $\lambda = 0$ and $n \to \infty$ it reduces to the normal.

The parameters of each model are estimated by maximum likelihood using the BHHH algorithm with a convergence criterion of 0.0001 applied to the log likelihood function value. In order to restrict the skewness parameter, λ , to lie in its valid range of -1 to +1, the following logistic transformation was used

$$\lambda^* = \frac{2}{1 + e^{-\lambda}} - 1$$

Data

- Daily price observations for five equity market indices [FT All Share, S&P500, Japan, Germany, Canada (Topix)] and a World equity market index
- Data source is **Datastream** (code PI) for the maximum available for each series
- For UK, US and Japan 01/01/1969-31/12/1999 (**n=8088**)
- For World equity market Canada and Germany 01/01/1973-31/12/1999 (n=7045)
- Continuously compounded returns are calculated as the first difference of the natural logarithm of each series, r_t=lnI_t-lnI_{t-1}
- Descriptive statistics reported as follows

Table	1.	Preliminary	statistics
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	US	UK	Japan	Canada	Germany	World
Sample period	01/01/69- 31/12/99	01/01/69- 31/12/99	01/01/69- 31/12/99	01/01/73- 31/12/99	01/01/73- 31/12/99	01/01/73- 31/12/99
Number of observations	8088	8088	8088	7045	7045	7045
Mean	3.3×10^{-4}	3.6×10^{-4}	3.2×10^{-4}	4.4×10^{-4}	4.1×10^{-4}	4.7×10^{-4}
Variance	8.9×10^{-5}	9.7×10^{-5}	9.3×10^{-5}	6.1×10^{-5}	8.3×10^{-5}	4.9×10^{-5}
S. skewness	-68.56	-11.15	-19.03	-30.16	-30.37	-20.57
S. kurtosis	877.43	181.70	310.87	305.54	195.96	222.69
Jarque-Bera	774 590.70	33 139.56	97 006.72	94267.45	39 320.62	50 01 1.93

Notes: The table reports preliminary statistics for each of the six return series. Under the null hypothesis, the standardized skewness and kurtosis statistics have a standard normal distribution. Under the null hypothesis, the Jarque–Bera statistic has a chi-squared distribution with two degrees of freedom. The 1% critical value is 9.21.

Does excluding extremes matter? Convergence problem leads to a trade-off between realistic risk measurement and getting desirable results.

	US	UK.	Japan	Canada	Germany	World
Mean equation						
α ₀	0.0003 (0.0000)	0.0004 (0.0000)	0.0003 (0.0000)	0.0004 (0.0000)	0.0004 (0.0000)	0.0004 (0.0000)
α1	0.0840 (0.0110)	0.1683 (0.0113)	0.0793 (0.0108)	0.2309 (0.0114)	0.0991 (0.0121)	0.2308 (0.0122)
α_2	_	_	0.0134 (0.0104)			-0.0241 (0.0122)
Variance constion						
β_0	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
β_1	0.0559 (0.0049)	0.0888 (0.0067)	0.1323 (0.0099)	0.0825 (0.0075)	0.0754 (0.0067)	0.0794 (0.0067)
β_2	0.9343 (0.0056)	0.8903 (0.0075)	0.8639 (0.0087)	0.8958 (0.0087)	0.9135 (0.0069)	0.9041 (0.0074)
k	1.6519 (0.0701)	2.4515 (0.1207)	1.4254 (0.0662)	1.9124 (0.1037)	2.0243 (0.0927)	2.0783 (0.1132)
п	10.6003 (1.4975)	7.1635 (0.6572)	7.7237 (1.1130)	6.2576 (0.7555)	7.1135 (0.6181)	7.8458 (1.0550)
Å	-0.0602 (0.0146)	-0.1674 (0.0148)	-0.0482 (0.0123)	-0.0947 (0.0154)	-0.1115 (0.0160)	-0.1489 (0.0159)
S. skewness S. kurtosis	-3.37 87.04	-6.90 75.11	-3.89 128.52	-5.71 102.71	-5.58 83.88	-6.30 162.27
LnL	27 548.24	27345.47	28 149.20	25 629.57	24 441.89	26057.92
Student-t LnL LR	27 537.80 20.88	27 323.78 43.38	28 125.27 47.85	25624.26 10.64	24 435.14 13.50	26046.87 22.10
Power exp. LnL LP	27 529.24	27 271.85	28 134.53	25 590.48	24385.25	26021.37
Normal LnL	27 286.44	27 167.92	27 572.04	25 340.76	24091.74	25916.70
LR	523.60	355.10	1154.31	577.64	700.30	282.44

Table 2. AR-GARCH estimates with SGT conditional distribution

Notes: The first panel gives the estimated parameters, with standard errors in parentheses, and maximum log likelihood (lnL) of the AR(p)-GARCH(1,1) model with an SGT conditional distribution. The second panel gives the maximum log likelihood value for each of the three distributions, Student-t, power exponential and normal, and the likelihood ratio statistic (LR) to test the restrictions on the SGT that they imply. The 1% critical values for the LR statistic are 9.21 for the Student-t and power exponential distributions and 11.34 for the normal distribution.

	US	UK.	Japan	Canada	Germany	World
Mean equation						
a ₀ .	0.0003	0.0003	0.0003	0.0004	0.0004	0.0003
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
α_1	0.0864	0.1701	0.0803	0.2319	0.0969	0.2370
	(0.0107)	(0.0110)	(0.0104)	(0.0113)	(0.0117)	(0.0118)
<i>a</i> ₂	_	_	0.0164	—		-0.0167
			(0.0101)			(0.0119)
Variance equation						
β_0	0.2010	-0.3286	-0.4125	-0.3403	-0.2243	-0.3030
	(0.0230)	(0.0296)	(0.0360)	(0.0397)	(0.0256)	(0.0338)
β_1	0.1081	0.1792	0.2250	0.1686	0.1424	0.1447
•	(0.0093)	(0.0109)	(0.0139)	(0.0127)	(0.0106)	(0.0111)
β_2	0.9876	0.9805	0.9740	0.9788	0.9882	0.9814
	(0.0021)	(0.0027)	(0.0032)	(0.0035)	(0.0022)	(0.0030)
v	-0.4651	-0.1079	-0.3546	-0.1189	-0.0870	-0.3455
	(0.0741)	(0.0433)	(0.0423)	(0.0514)	(0.0495)	(0.0591)
k	1.6643	2.3868	1.4203	1.9072	2.0343	2.1103
	(0.0704)	(0.1164)	(0.0627)	(0.1036)	(0.0939)	(0.1169)
п	10.5345	7.3393	7.8508	6.2413	7.3232	7.6491
	(1.4700)	(0.6925)	(1.0567)	(0.7492)	(0.6118)	(1.0255)
à	-0.0049	-0.1315	0.0282	-0.0733	-0.1074	-0.0532
	(0.0298)	(0.0334)	(0.02.58)	(0.0321)	(0.0334)	(0.0341)
S. skewness	-0.37	-5.05	-1.10	-2.40	-4.79	-2.39
S. kurtosis	91.78	82.60	115.65	106.22	89.94	85.66
LnL	27584.02	27 3 32.21	28218.20	25 622.17	24441.09	26086.92
Student-t						
LnL	27 577.92	27320.67	28 197.80	25 61 9.13	24435.78	26076.30
LR	12.20	23.08	40.80	6.08	10.62	21.24
Power exp.						
LnL	27 566.60	27 272.96	28 198.61	25 589.70	24390.56	26059.92
LR	34.84	118.50	39.18	64.94	101.06	54.00
Normal						
LnL	27338.66	27175.24	27632.14	25343.63	24105.91	25968.13
LR	490.72	313.94	1172.12	557.08	670.36	237.58

Table 3. AR-EGARCH estimates with SGT conditional distribution

Notes: The first panel gives the estimated parameters, with standard errors in parentheses, and maximum log likelihood (lnL) of the AR(p)-EGARCH(1,1) model with an SGT conditional distribution. The second panel gives the maximum log likelihood value for each of the three distributions, Student-t, power exponential and normal, and the likelihood ratio statistic (LR) to test the restrictions on the '38 SGT that they imply. The 1% critical values for the LR statistic are 9.21 for the Student-t and power exponential distributions and 11.34 for the normal distribution.

Results

 This study has shown that the use of the SGT conditional distribution offers a substantial improvement over the normal, Student-t and power exponential distributions that are typically used for modelling the conditional volatility of daily equity returns.

Results (Cont'd)

- Conditional distribution of returns is negatively skewed for all six series (GARCH-SGT).
- Skewness in USA, Japan and the World index can be explained by asymmetry in the response of volatility to return shocks, and is captured by the EGARCH-SGT model.
- Correct specification of the conditional distribution of returns is important for financial risk management and VaR.
- VaR is very sensitive to existence of significant skewness and kurtosis in the distribution.
- VaR of a portfolio will be larger, the more negative the skewness of the conditional distribution of portfolio returns and the greater its kurtosis.

Results (Cont'd)

- Correct specification of the conditional distribution of returns is also potentially important for asset pricing.
- Correct specification of the conditional distribution of returns is particularly important for option valuation, where the widely used Black-Scholes model – which relies on the assumption of lognormality – generally mis-prices options that are deep in-themoney or deep out-of-themoney (see Hull, 2000).
- Corrado and Su (1996) compute the implied skewness and kurtosis of option prices and show that allowing for non-normality improves the accuracy of the Black–Scholes model.

- Research Limitations / Future Implications
 - Commodity markets' returns (energy –oil, natural gas, electricity-, precious metals) needs to be included in the analyses together with financial asset returns (including high frequency data).
 - New algorithms apart from BFGS and BHHH might offer opportunities for fast and efficient results. As Chris Brooks investigated through his several papers both algorithms and econometric/statistics packages might produce different results (danger! If you're running a big portfolio and relying solely on results!).

- Research Limitations / Future Implications (cont'd)
 - Although burdensome and problematic, distribution of returns needs to be chosen carefully.
 - Do not rely on mean-variance only! Higher moments worth to be taken into account.
 - Your choice of the distribution and hence the model has a severe impact on your risk measures and companies' risk profile.
 - Concrete evidence from real life portfolios needed to verify the necessity of the use of these models.
 - Incorporating model outputs into risk management decision process requires expertise, intuition and experience.
 - Basel-II...?

- Evidence for Future Implications (1)
 - Anson et al (2007) "Building a Hedge Fund Portfolio with Kurtosis and Skewness", in J of Alternative Investments
 - Hedge fund return distributions are distinctly non-normal.
 - Their return patterns display significant skewness and leptokurtosis.
 - As a results standard mean-variance optimization may produce inefficient portfolios.
 - To correct this problem, they apply a four-moment analysis to a "live" portfolio of hedge funds.
 - They show that using all four moments of the return distribution in optimization they get higher cumulative performance, a less negative skewness and less volatility.
 - ATTENTION: This approach does not eliminate outlier *event risk*

- Evidence for Future Implications (2)
 - Dimou et al (2005) "Skewness of Returns, Capital Adequacy, and Mortgage Lending", in J of Financial Services Research
 - They calibrate a simulation model of credit value-at-risk for mortgage lending to UK experience.
 - Simulations to capture the skewness of returns that might arise in the context of a financial crisis suggest that the IRB calculations of the <u>new Basel Accord can substantially understate prudential</u> <u>capital adequacy</u>.
 - The same model shows that raising capital requirements has only a small impact on bank funding costs.
 - They conclude that Pillar 2 supervisory review should increase capital requirements above IRB levels for secured bank assets those whose returns can potentially fall furthest, relative to other, normally "riskier" assets, in extreme outcomes.

- Evidence for Future Implications (3)
 - Rachev et al (2005) "Equity and Bond Return Distributions", in *Fat Tailed and Skewed Asset Return Distributions*, Ch.11.
 - The US Agency Mortgage Passthrough Securities
 - They are issued by Government National Mortgage Association (Ginnie Mea), the Federal Home Loan Mortgage (Freddie Mac) and the Federal National Mortgage Association (Fannie Mea).
 - The agency mortgage passthrough securities sector is included in the broad based bond market indices created by Lehman Brothers, Salomon Smith Barney and Merrill Lynch.
 - Lehman Brothers labels this sector of its bond market index the "mortgage passthrough sector".
 - 40% of the whole sector is represented by Lehman Brothers Aggregate Bond Index.

- Evidence for Future Implications (3)-cont'd
 - Rachev et al (2005) "Equity and Bond Return Distributions", in *Fat Tailed and Skewed Asset Return Distributions*, Ch.11.
 - These securities portfolios are large and must be hedged.
 - For example 22 dealers' securities portfolio had reached to \$40 billion in October 2004.
 - The creators of bond indices do not include all of the pools in the market.
 - Instead they create composites of these securities, what LEhman Brothers refers to as *index generics*
 - Without a firm understanding of the return distribution properties of these securities, dealers cannot adequately hedge positions.



Thanks for your patience

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Now it's time for Q&A

(Correct) Answers are <u>not</u> guaranteed!

Difficult questions are preferred and welcome ONLY!