AN ASSESSMENT OF VALUE-AT RISK (VaR) AND EXPECTED TAIL LOSS (ETL) UNDER A STRESS TESTING FRAMEWORK FOR TURKISH STOCK MARKET

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Abstract

This paper investigates various methods of Value-at-Risk (VaR) and Expected Tail Loss (ETL) estimation for the Turkish stock market under a stress-testing framework, in which implausible but probable scenarios are generated by the three major economic downturns that affected Turkey in 1994, 1998 and 2001. For each period and for each method, out of sample forecasts are backtested via Basel's backtesting procedure and Kupiec's Proportion of Failure Test.

The results show that ETL is a superior risk measure than VaR, additionally among the methods used for variance-covariance methodology, the best performance is achieved when conditional volatility is modeled as GARCH (1,1) with Generalized Error Distribution. Moreover, as a non-parametric approach, Filtered Historical Simulation (FHS) method performs even better than some parametric methods

Key words: Value at Risk, Expected Tail Loss, Stress Testing, Backtesting, Forecasting
JEL Classification: G10, C22, C53

1. INTRODUCTION AND LITERATURE REVIEW

Financial risk is the viewpoint of financial losses and gains, which can be conceptualized by unanticipated developments that governs the risk factors. Financial risk is characterized by various components such as market risk, credit risk, operational risk, liquidity risk, strategic risk, etc. In this paper, we are only concerned with the market risk, which can be defined as "risk of loss (gain) arising from the unexpected changes in market prices of the underlying assets" (Dowd, 2002). Fundamentally, market risk can be decomposed into various sub-risk components such as interest rate risk, foreign exchange risk, equity price risk, etc. depending on which risk factors affect the sub-risk component of the market risk. One or more of the sub-risk components can affect the risk of the underlying asset. For example, a portfolio composed of only common stock might be exposed to only equity price risk; however, a portfolio composed of Eurobond derivatives might be exposed to both interest rate and foreign exchange risks as a market risk.

As the rapid development of the financial markets interrupted by the major financial breakdowns during the last decade, risk management, particularly market risk management, has become increasingly important for both market participants and market regulators. They need models for measuring, managing and controlling the market risk. Market participants, such as investors and banks need risk measurement management models to deal with the risks involved in their market positions and their undergoing operations. On the other hand, market regulators need these

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models to ensure the stability and integrity of financial markets by setting requirements such as minimum capital requirement in the case of banking industry or setting margin requirements in the case of the stock exchanges. Particularly, the 1988 Basel Accord, 1996 Basel Amendment and lastly Basel II proposals have given an impetus in measuring and controlling financial risk for both regulators and market participants[1, 2].

One of the most popular models used for market risk management is Value-at- Risk (VaR), popularized by the JP Morgan RiskMetrics TM [3]. In the most basic definition, VaR can be defined as the maximum amount of money we are likely to lose over a pre-specified period, for a specific confidence level. But as it is evident from the definition, VaR does not tell about if a bad event, or tail event, (beyond confidence level) occurs. It just states the most we can expect to lose if a bad event (beyond confidence level) does ***not*** occur.

More formally, VaR can be defined as,

$$VaR_t(\alpha) = \mu_t + F_t^{-1}(\alpha)\sigma_t \qquad \textbf{(1)}$$

or interchangeably, VaR can be defined as the quantile,

$$\Pr(r_t \leq VaR_t(\alpha)) = \alpha \qquad \textbf{(2)}$$

$$r_t = \mu_t + \varepsilon_t \qquad \textbf{(3)}$$

where, σ_t and μ_t are the mean and the standard deviation of returns at time *t*, F_t^{-1} is the inverse of the distribution of the return process given by (3) and α is the confidence level that we choose.

Although its popularity and ability to quantify the risk as a single measure, VaR is criticized in several grounds, from its underlying statistical assumptions to the way how it is used. For instance, Nassim Taleb (1997) and Richard Hoppe (1998, 1999) criticized the use of statistical approaches of physical sciences in financial risk management, especially in VaR, since risk management has an inherent nature of social systems that does not coincide with the physical systems. Another argument against the VaR is that different VaR models can give very different estimates, which in turn makes VaR estimates imprecise (Beder T., 1995, Marshall and Siegel 1997). The implementation of VaR as a regulatory risk requirement also gets severe critics in such a way that it destabilizes the financial system if all the market participants act according to the regulatory VaR. (Danielson (2002), Taleb (1997a, b)). This phenomenon is also called counterproductive effect of VaR.

Another rigorous criticism of VaR comes from the question whether it is the best tail-based risk measure. What we mean by the "tail-based risk measure" is that it is a risk measure based on the probability distribution of tails. Apparently, VaR is not the best tail-based risk measure. Work by Artzner et.al. (1999) titled "Coherent Measures of Risk" reveals that VaR is not a good measure of risk in terms of axioms they had proposed. Actually, what they had showed is that VaR did not satisfy the sub-additivity axiom, which states a coherent risk measure of combined positions should be smaller than the sum of individual positions coherent risk measure. More formally, what Artzner et al.(1997,1999) put forward is that a "Coherent Risk Measure" satisfy the below axioms for all random values of two risky position *X* and *Y* and risk measure $\rho(.)$

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[1] Discussion of market risk from the regulatory perspective can be found in "Amendment to Capital Accord to Incorporate Market Risks", Basel Committee on Banking Supervision (1996), BIS.

[2] Regulatory evolution of VaR models is discussed in Lopez. J.A (1999).

[3] Historical perspective of VaR and how it evolves can be found in RiskMetrics Technical Development and books such as Measuring Market Risk (2002), Beyond Value at Risk (1998) by Dowd K. and Value-at Risk: The new benchmark for controlling market risk (1997) by Jorion P., Market Models: A guide to financial data analysis(2001) by Alexander C.

$$\rho(X+Y) \leq \rho(X)+\rho(Y) \quad \text{(sub-additivity)} \qquad \textbf{(4)}$$

$$\rho(X) \leq 0, if \ \ X \geq 0 \quad \text{(monotonicity)} \qquad \textbf{(5)}$$

$$\rho(tX) = t\rho(X), t \geq 0 \quad \text{(homogeneity)} \qquad \textbf{(6)}$$

$$\rho(X+n) = \rho(X) - n \quad \text{(translation invariant)} \qquad \textbf{(7)}$$

The condition given by (4) is the simply the sub-additivity property discussed above and the conditions given by (5) and (6) is natural in the way we define a risk measure. Last condition given by (7) simply states that addition of risk free (sure) amount to our position would decrease the risk.

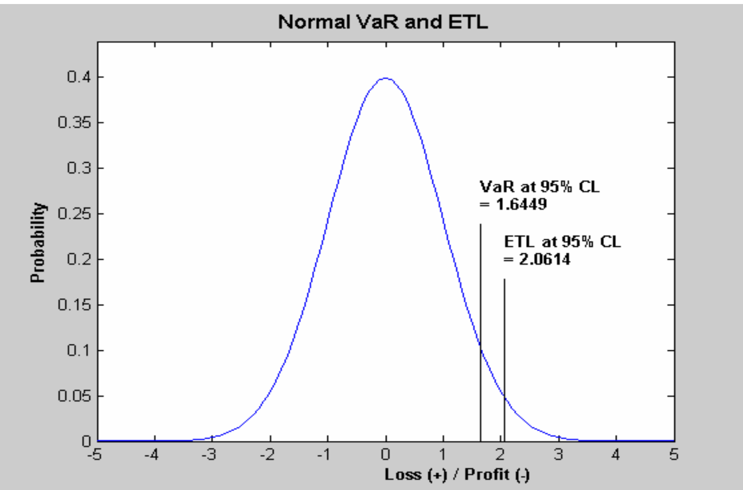
Enlighten by the above discussion, the first question that comes to mind is whether there is a coherent risk measure, which is easily implemented as VaR. The answer comes from again Artzner et.al. They show that any coherent risk measure can be regarded as the maximum expected loss on a set of "generalized scenarios" where a generalized scenario is a set of loss values and their associated probabilities (Artzner et al, 1999). The proposed coherent risk measure in terms of what Artzner et al.(1997,1999) suggest, can be Expected Tail Losses (ETL), which can be described as the expected value of the losses (losses on a set of generalized scenarios) that is governed by particular distribution such as normal distribution.

ETL can be characterized by different names in the literature, which some of them are expected shortfall (ES), Conditional VaR (CVaR) and Tail Value-at-Risk (TVaR).[4] The formal definition of ETL can be given by[5]

$$ETL = ETL[Loss \mid Loss > VaR] \qquad \textbf{(8)}$$

As it is evident from (8), ETL says that what we can expect to lose if a bad event (tail event) occurs on the average, in contrast to traditional VaR models' inability to tackle the risk beyond d the confidence level. A visualization of difference between VaR and ETL is given in Figure 1, which shows the ETL model's ability of tackling with tail events.

Figure-1: Expected Tail Loss vs. VaR at 95 % Confidence Level of Standard Normal Distribution.[6] (Losses are given as positive value)



[4] For a complete discussion of Expected Tail Loss and its variants, you can refer to Acerbi and Tasche (2002).
[5] Following Dowd K. (2002), Measuring Market Risk , p. 32.
[6] Figure 1 is produced by the Matlab Toolbox supplied by Dowd. K (2002), Measuring Market Risk.

Several methods have been suggested to calculate VaR and ETL measures under various assumptions. As a result of this variety, VaR and ETL models differ vastly in the outcome, i.e. VaR or ETL number. This also makes for the market participants and regulators difficult to choose a method, which appropriately measures the market risk. Eventually, as suggested by Beder (1995), this makes VaR estimates imprecise and this increase the market participants' exposure to the implementation risk as well (Marshall and Siegel, 1997). The implementation risk of VaR and ETL models coupled with the wide selection of such models necessitate careful examination of these models by referring and testing their underlying assumptions under the stress testing and backtesting frameworks. Actually, the examination and validation of such models in emerging market setting is very crucial because of the fact that the high volatility, resulting in increase of the major risk factors, have been produced by the economic and financial crisis in the emerging markets. Crisis in Asia in 1997-1998, in Russia in 1998 and in Latin America and Turkey in 2001 can be given as good examples of above argument. A good summary for emerging market crisis can be found in the article "A primer on emerging market crisis" by Dornbush, R. (2001).Also, In a working paper, Danielson and Saltoglu (2003) analyzed the financial crisis in Turkey in terms of market microstructure methods and concluded that role of financial markets in crises occurring in emerging markets should not be ignored by supervisory and supra-national organizations.

Turkey can be given as a good example of emerging markets both in terms of its market economy structure and crises that she faces during the last 15 years. Particularly, the economic crisis in 1994 and in 2001 and the spillover effects of Asian, Russian and Latin American crises had a deep impact on the financial institutions in Turkey especially for the banking industry. Eventually, these financial disasters increased the awareness of risk management practices. However, only few studies can be seen on evaluating the risk forecasts of various models for the Turkish economy despite the significant need and interest in both academic and regulatory purposes. A few exceptions are Gencay and Selçuk (2004) work, which analyzed the Turkish stock market besides major emerging markets in the context of Extreme Value Theory, and Harris and Küçüközmen (2001) work, which shows the implications of linear and non-linear dependence on VaR calculations.

The objective of this paper is to test VaR and ETL figures generated by certain VAR methods in the context of Turkish stock market. The aim here is to compare these models by various backtesting procedures in a stress-testing framework. Stress testing procedure can be characterized as testing the models in-sample and out-of-sample forecast periods, which corresponds to the periods of three major economic breakdowns (1994,1998,2001), (the out-of sample periods gives us the possible scenarios). For a backtesting procedure, Basel proposed backtesting -Traffic Light Test-, Kupiec tests (Kupiec, 1995) are used.

The organization of the paper is as follows. In Section 2, various VaR and ETL methodologies are discussed. In Section 3 the forecast procedure and backtesting procedures are discussed and Section 4 presents the empirical results and conclusions.

2. VALUE AT RISK AND EXPECTED TAIL LOSS METHODOLOGIES

2.1 Definition of VaR and ETL

Let the returns of an asset at time *t* is given by (9) where p_t is the price of an asset at time *t*.

$$r_t = \log(p_t / p_{t-1}) \qquad \textbf{(9)}$$

The $\boldsymbol{VaR_t(\alpha)}$ at the $(1\text{-}\alpha)$ percentile (confidence level) can be defined by,

$$\Pr(r_t \leq VaR_t(\alpha)) = \alpha \qquad \textbf{(10)}$$

In the most basic definition, as it is evident from (10), VaR can be defined as the maximum amount of money we are likely to lose over a pre-specified period, for a specific confidence level $1-\alpha$.

However, ETL is defined as the expected loss given that the loss is excess of the specified VaR value in confidence level $1-\alpha$ at time t. As it is evident from the Figure 1 and the Equation (11), ETL gives us a more conservative risk measure.

$$ETL_t = E[L_t \mid L_t > VaR_t(\alpha)] \qquad \textbf{(11)}$$

We now turn to various methods of estimating the $VaR_t(\alpha)$ and ETL_t. We classify the models into four main categories. (1) Variance-Covariance methods (2) Non-parametric methods (3) Monte Carlo Methods (4) Extreme Value Theory Based VaR and ETL methods. In the paper from now on, estimation procedures for VaR first discussed and it is shown that how easily ETL estimates can be extracted by averaging the tail VaRs. The method (taking average tail VaRs to obtain ETL) will be discussed thoroughly in section Variance-Covariance Methods.

2.2 Variance-Covariance Methods

2.2.1 VaR Estimation

The variance-covariance method is the most basic approach among various models used to estimate VaR. In our paper, however, since we deal with single stock index instead of portfolio, we do not consider covariances. In this method $VaR_t(\alpha)$ can be estimated by (1).

$$VaR_t(\alpha) = \mu_t + F_t^{-1}(\alpha)\sigma_t$$

where, μ_t and σ_t are the mean and the standard deviation of returns at time t respectively and F_t^{-1} is the inverse of the distribution of the return process given by (3) and α is the confidence level that we choose. Apparently, estimation of VaR_t occupies the estimation of $F_t(.)$, μ_t and σ_t. We can consider different methods of estimating these values. A simplest one is using the **sample mean and sample variance (unconditional volatility)**. Sample variance and sample mean can be obtained by

$$\mu_t^s = (\sum r_i)/n \qquad \textbf{(12)}$$

$$\sigma_t^2 = (\sum [r_i - \mu_t^s])/(n-1) \qquad \textbf{(13)}$$

Despite its simplicity, unconditional volatility has some major drawbacks. For example, it does not take into account any information about the intraday volatility. Again it weights the past returns equally and assumes that the returns are normally distributed, which is an unrealistic assumption for financial time series. Actually financial time series exhibits deviations from normality, which is one of the most important stylized facts that characterize the stock returns (Cont, 2001). Also this method may overestimate or underestimate the volatility because of the fact that it weights the returns equally, which in turn, is against the heteroscedastic nature of financial returns.

A possible solution to the above mentioned problems are to estimate the sample variance by a conditional volatility model. The most basic conditional volatility model is **EWMA (Exponentially Weighted Moving Average)**, which is popularized by the RiskMetrics™. EWMA is a method to calculate the variances of the return series by weighing the latest observations higher than distant past data. By doing so, EWMA keeps a memory of its recent past. Variance is calculated from EWMA as follows, where r_t denotes the return at time t.

$$\sigma_t^2 = (1-\lambda)\sum_{j=0}^{\infty} \lambda^{j-1}(r_{t-j} - \bar{r})^2 \qquad \textbf{(14)}$$

RiskMetrics^{TM} methodology sets λ=0.94 for the decay factor, which indicates how much weight is given to recent past and distant past.

The most promising conditional volatility estimation model and what we use as a variance estimation model for variance-covariance methods in this paper, is the **GARCH type volatility models**, which were developed by Engle (1982) and Bollersev (1986). GARCH (p, q) model is given by

$$y_t = c + \varepsilon_t$$
$$\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2 \qquad \textbf{(15)}$$

where y_t is the return process and the ε_t is the residuals. In the paper, we evaluate volatility estimations from GARCH (1,1) under the assumption that ε_t is normally distributed, student-t distributed and GED distributed.

Given the leptokurtic distribution property of financial time series, it may be desirable to use a distribution for ε_t, which has fatter tails than the normal distribution. The student-t and generalized error distributions are such distributions to handle these fat tails. We also estimate the degrees of freedom parameter of student–t distribution and parameter "v" of GED that governs the thickness of tails of the distribution. Probability density functions of student-t and GED distributions are given below (Kendall, Stuart, 1972)).

Student-t distribution

$$f(u_t) = \frac{\Gamma\left[(v+1)/2\right]}{(\pi v)^{1/2}\,\Gamma(v/2)} \frac{s_t^{-1/2}}{\left[1 + u_t^2/(s_t v)\right]^{(v+1)/2}} \qquad \textbf{(16)}$$

, where Γ is the gamma function and v is the *d.o.f.* parameter

Generalized Error Distribution

$$f(u_t) = \frac{v \exp\left[-(1/2)\,|u_t/\lambda|^v\right]}{\lambda.2^{(v+1)/v}\,\Gamma(1/v)}$$

where **(17)**

$$\lambda = \left[\frac{2^{-2/v}\,\Gamma(1/v)}{\Gamma(3/v)}\right]^{1/2}$$

and v is a positive parameter governing the thickness of the tail behavior of the distribution.

2.2.2 ETL Estimation

Since we define the ETL as the probability weighted average of tail losses, or losses exceeding VaR (**11**), ETL can be estimated as an average of tail VaRs. The easiest way to way to implement this approach according to Dowd (2002:45) is to slice the tail into large number "n" of slices, each of which has the same probability distribution, and to estimate the VaR associated with each slice and to take the ETL as the average of these VaRs.

Dowd(2002) also gives an illustration of how to estimate an ETL at the 95% confidence level of a standard normal distribution with mean 0 and standard deviation 1, which is also given here in Table 1 to facilitate the ETL estimation procedure that is used in this paper.

Table -1: Estimating ETL as a weighted average of tail VaRs

Tail VaRs α	Tail VaR
At 95.5	1.6954
At 96.0	1.7507
At 96.5	1.8119
At 97.0	1.8808
At 97.5	1.9600
At 98.0	2.0537
At 98.5	2.1701
At 99.0	2.3263
At 99.5	2.5758
Average of Tail VARs	**1.9870**

In this procedure we should, of course use large n (slicing the tail in more small pieces) to be able to approach the true value of the ETL estimate. As it is seen from the table 1, if we use n=10, ETL estimate is 1.9870, however if we use n larger than 1000, ETL approaches the true value 2.061 as depicted in Figure 1.

2.3 Non-parametric Methods

All non-parametric approaches seek to estimate VaR and ETL without making strong assumptions about the distributions of returns given by (3). In other words, we use the empirical distributions rather than theoretical distribution to estimate the VaR and ETL. The first non-parametric approach that we use in this paper is simply Historical Simulation (HS), which is a histogram-based approach. The other method that we follow during our research is Filtered Historical Simulation (FHS) proposed by Barone-Adesi, Giannopoulaos and Vosper (1999, 2000). The other methods that can be employed as non-parametric VaR and ETL estimation are bootstrapping, non-parametric density estimations such as kernels and non-parametric quantile regression.

2.3.1 Historical Simulation

The main idea behind the historical simulation approach is the assumption that historical distribution of returns will remain the same over the next periods; therefore, the empirical distributions of portfolio returns will be used in estimation of VaR. In other words, the historical simulation method estimates the quantiles of an underlying distribution from the realization of the distribution. In this case HS is estimated by

$$VaR_t(\alpha) = \mu_t + F_t^{-1}(\alpha)r \qquad \textbf{(18)}$$

where $F_t^{-1}(\alpha)\, r$ is the q^{th} quantile of $(q=1-\alpha)$ of the sample distribution.

Although its simplicity in implementation, it has some major drawbacks. One of the most important is that it allocates same weights to all past returns. Therefore it is not able to tackle with the heteroscedastic nature of the financial returns. In other words, HS is slow to respond the shifts that take place during chosen sample period.

ETL estimates using basic HS has the same underlying concept as we use while estimating it by variance-covariance method. That is, again we use the average of tail VaRs method. To make it concrete, consider the example of estimating ETL at 95% confidence level from the historical return series of 100 observations. Obviously, VaR at 95% confidence level is the sixth highest losses of this empirical distribution, Therefore, average tail VaR method for estimating ETL suggests that the ETL at 95 % confidence level is the average of the five highest losses of our empirical distribution.

2.3.2 Filtered Historical Simulation

As we have mentioned, basic HS approaches fail to capture the conditionally varying volatilities. The most basic procedure to capture conditional volatility is using GARCH type models. Filtered Historical Simulation (FHS) preserves the non-parametric nature of HS by bootstrapping returns within a conditional volatility framework by combining the flexibility of bootstrapping and power of the GARCH models to tackle for problems arising from the leptokurtic and heteroscedastic nature of the financial returns. Here, we give a rather practical implementation of FHS in an algorithmic way. A complete FHS methodology is given in Barone-Adesi, Giannopoulaos and Vosper (1999).

First of all, we fit GARCH model to our stock index set. Use of a GARCH model, which is capable of extracting volatility clustering and leverage effects, is recommended by Barone-Adesi et al. (1999). Hence we decide to use EGARCH (1,1). Secondly, the volatility forecasts of the sample period are calculated to come up with the standardized returns. Then, assuming a 1-day holding period, we apply the bootstrapping approach to our set of standardized returns, which then multiply the each random drawing with the tomorrow's forecast of volatility. By doing so, we get simulated returns, which reflects the today's forecast of tomorrow volatility. And finally, each simulated returns gives us a possible path of losses, which we can easily implement the traditional VaR estimate for the chosen confidence level.

3. Forecasting Procedure and Backtesting

3.1 Data and Forecasting Procedure

The data used in this paper constitutes the Turkish stock market price index. The period for the stock price indices is between 01.01.1990 and 31.12.2000, which makes 10 years (2610 days) of observation of stock prices. Raw data are obtained from DataStream databases. We used the log return of stock prices, which is an appropriate choice if the concern of the research is about time series modeling such as GARCH modeling due to its time additivity. (Dornfleitner, 2003). The preliminary findings such as mean, standard deviation, third and fourth moments, about the stock return series can be found in the section of empirical results of the paper.

We compare the VaR and ETL models by their one step predictive performance in terms of out-of –sample forecast ability. To investigate the performance of VaR and ETL models under different circumstances, we use three out of sample evaluation periods, which corresponds to the major economic crisis periods (1994,1998,2001) (see Table 2) This procedure can also be regarded as a "***stress testing***" under different scenarios following Bao, Lee, Saltoglu (2006).

Table -2 : Out-of Sample Forecast Periods.

	Period 1 (1994 Crisis)	**Period 2 (1998- Russian Crisis)**	**Period 3 (2001 Crisis)**
Out-of-sample Period	1/1/1994-12/31/1994 P=261	1/1/1998-31/12/1998 P=261	1/1/2001-31/12/2001 P=261
In-sample Period	1/1/1990-31/12/1993 R=1045	1/1/1990-31/12/1997 R=2088	1/1/1990-31/12/2000 R=2870

For the forecasting procedure, we use a rolling window design. In a rolling window design, we have sample of total T observations and we divide it into an in-sample part of size R and out-sample part of size P so that T = P+R. And for the $(t\text{-}R)^{th}$ prediction is based on observations t-R to t-1, where t= R+1,…., T.

3.2 Backtesting

Backtesting refers to the tests that are conducted for determining the performance of the models when they are subject to different conditions. In other words, it answers the question that which particular risk model is suitable. In this paper, we mostly deal with the two type of backtesting procedure; one is the typical regulatory backtesting requirement proposed by Basel Committee, namely Basel Traffic Light Test (1996), the other is the backtesting procedures that are based on statistical properties of tail losses, namely Kupiec (1995) test. Other popular backtesting procedures are Christoffersen (1998) approach and Crnkovic-Drachman Backtest Procedure(1995).

3.2.1 Basel Proposed Backtesting -Traffic Light Test -

The Basel Traffic Light test only considers the number of exceptions and puts the results in to three zones, green, yellow, and red. The classifications are based on 99% confidence level for 250 VaR figures (approximately one year). Green zone tolerates up to 4 exceptions, whereas yellow zone tolerates 5 to 8 exceptions. However, if numbers of exceptions are above 8, the VaR models are taken to be in red zone. A method, which is placed in green zone, is said to perform adequately. The one in the yellow zone is acceptable but further research is needed. But if a method is in red zone, it should be replaced.

3.2.2 Kupiec Tests

Kupiec tests are so-called "Likelihood-Ratio Tests". They check how similar two quantities are by calculating a statistic based on their ratio. These test tests the hypothesis that the empirical probability is equal to the theoretical probability. In other words,

$$H_0 : p^e = p^t$$

where p^e is empirical probability and p^t is the theoretical probability. The ideal value for the likelihood ratio(LR) is zero. Actually this will happen when the two-probability figure is equal. In Kupiec's Proportion of Failures (POF) Test we check the proportion of failures or exceptions. This proportion is similar to significance level that we use when computing VaR. For example, at 99% confidence level null hypothesis become

$$H_0 : p^e = p^t = 0.01 = x / n$$

where *x* is the number of exceptions and *n* is the sample size. The likelihood function is of this test is asymptotically χ^2 distributed with d.o.f of 1. If the value of the LR exceeds critical value, we will reject the null hypothesis. Another Kupiec's tests are the time until first Failure Test (TUFF-test) and mixed Kupiec tests proposed by Kupiec (1995).

4. EMPRICAL RESULTS

4.1 Preliminary Statistics

Our preliminary findings (first four moments and normality test statistic of the series), which are shown in the Table 3, suggest that in Turkish stock market, there is a significant excess kurtosis and deviation from the normality, which can also be evidenced by large value of normality test statistics of Jarque- Bera. This stylized fact is consistent with the characterization of the financial markets, namely leptokurtic, fat tailed distribution mentioned in the financial literature.

Table-3 : Summary Statistics

Summary Statistics	
Mean	0.001862
Median	0
Maximum	0.170258
Minimum	-0.1946
Std. Dev.	0.030305
Skewness	-0.01845
Kurtosis	6.046203
Jarque-Bera*	1412.602
*** significant at 1%**	

4.2 Results for Basel Proposed Backtesting (Traffic Light Zone Test)

The Basel Traffic Light Zone Test is carried out as explained in the Section 3. The results at 99% confidence level are depicted in the Table 4. Several important conclusions can be drawn from the results of the test:

Table -4: Basel Traffic Light Test

Basel Traffic Light Test			
Traffic Light Zones at 250 Test Points at 99 % confidence level			
Green	Up to 4	Up to 4	Up to 4
Yellow	Up to 8	Up to 8	Up to 8
Red	Above 8	Above 8	Above 8
Methods (VaR)	**Period 1 (1994)**	**Period 2 (1998)**	**Period 3 (2001)**
Sample Variance	7	5	10
Traffic Light	Yellow	Yellow	Red
EWMA	6	6	10
Traffic Light	Yellow	Yellow	Red
GARCH (1,1)-normal	7	6	9
Traffic Light	Yellow	Yellow	Red
GARCH (1,1)-student-t	5	4	8
Traffic Light	Yellow	Green	Yellow
GARCH (1,1)-GED	3	4	5
Traffic Light	Green	Green	Yellow
HS	10	11	15
Traffic Light	Red	Red	Red
FHS- (EGARCH (1,1)-n)	6	5	8
Traffic Light	Yellow	Yellow	Yellow
Methods (ETL)	**Period 1 (1994)**	**Period 2 (1998)**	**Period 3 (2001)**
Sample Variance	4	3	8
Traffic Light	Green	Green	Yellow
EWMA	3	4	7
Traffic Light	Green	Green	Yellow
GARCH (1,1)-normal	4	5	6
Traffic Light	Green	Yellow	Yellow
GARCH (1,1)-student-t	3	2	5
Traffic Light	Green	Green	Yellow
GARCH (1,1)-GED	2	2	3
Traffic Light	Green	Green	Green
HS	7	8	8
Traffic Light	Yellow	Yellow	Yellow
FHS- (EGARCH (1,1)-n)	5	4	5
Traffic Light	Yellow	Green	Yellow

- In accordance with empirical evidence reported elsewhere, ETL estimates are superior than the VaR estimates as it is evident from the fact that in period 3, which is the most severe conditions of the stress testing, no ETL estimates is placed on Red Zone. Instead, GARCH (1,1)-GED estimate of ETL is in Green Light Zone, which is the only model that passes all periods with green light.

- GARCH models with student-t distribution and GED give more adequate estimates for both VaR and ETL. Actually, this is what we expect to find out since it is evident from the preliminary statistics that return series have leptokurtic distribution, i.e., fat tails.

- Compared with the HS, filtered-historical simulation method improves the estimation for both ETL and VaR.

4.3 Results for Kupiec Test

The results of the Kupiec's Test (proportion of failures) at 99% confidence level are depicted in the Table 5. Result of the confidence level of 99% is only given here to be able to compare these with Basel Traffic Light Test. Most of the conclusions drawn from Basel Traffic Light Test are also confirmed by the Kupiec's POF test. The results of the Kupiec's TUFF test is not given here since, Kupiec's TUFF test is inferior to the basic Kupiec's POF test. (Dowd, 2002). An important result that should be mentioned in here is that the rejection of null hypothesis that the theoretical probability (0.01) is equal to empirical probability (x/n) for methods that passed Basel Traffic Light Zone test at first glance seems controversial. However, since Kupiec's POF test equality of the probabilities, we may suspect that these models who rejects null hypothesis simply rejects because of the low exceptions, or in other words they may reject the null hypothesis since they overestimate the risk. GARCH (1,1)-GED model can be given as a good example of this phenomenon.

Table-5: Results of Kupiec Test

Kupiec Test (Proportion of Failures)			
χ^2 statistic at 1 degree of freedom at 99 % confidence level is 6.635			
Methods (VaR)	**Period 1 (1994)**	**Period 2 (1998)**	**Period 3 (2001)**
Sample Variance	**A**	**A**	**A**
EWMA	**A**	**A**	**A**
GARCH (1,1)-normal	**A**	**A**	**A**
GARCH (1,1)-student-t	**A**	**A**	**A**
GARCH (1,1)-GED	**N**	**N**	**N**
HS	**A**	**A**	**A**
FHS- (EGARCH (1,1)-n)	**A**	**A**	**A**
Methods (ETL)	**Period 1 (1994)**	**Period 2 (1998)**	**Period 3 (2001)**
Sample Variance	**A**	**N**	**A**
EWMA	**N**	**N**	**A**
GARCH (1,1)-normal	**N**	**A**	**A**
GARCH (1,1)-student-t	**N**	**N**	**A**
GARCH (1,1)-GED	**N**	**N**	**N**
HS	**A**	**A**	**A**
FHS- (EGARCH (1,1)-n)	**A**	**N**	**A**
A: Alternate Hypothesis N: Null Hypothesis			

5. CONCLUSION AND IMPLICATIONS FOR FURTHER RESEARCH

In this paper, a comprehensive predictive assessment of various VaR and ETL models (for Variance-Covariance and Historical simulation methods) are conducted for the Turkish stock market. We have made assessment of the seven VaR models and corresponding seven ETL models in a three "stress testing period". We have made the backtesting of these models in the Regulatory Backtesting Framework (Basel Traffic Light Zone Test), and popular Kupiec's POF test. A major finding is that ETL is superior to the VaR risk models. However, ETL models have the peril of overestimating the risk measures. Overestimated risk measure results in holding excess capital for banking industry, which in turn hinders the profitability of banks. On other hand, from a regulatory perspective, overestimating the risk for certain amount may be tolerable because of the fact that holding capital acts as a safety cushion for entire financial system in case of any market instability.

Another important finding is that among the Variance-Covariance Methods, GARCH type VaR estimation is better if the error terms of the GARCH process follow Generalized Error Distribution (GED). Actually, this is what we should expect given that the fat-tailed nature of the financial return series. Also, as a non-parametric method, filtered historical simulation performs equally or better than some of the parametric methods. This may have implications for assessing the VaR or ETL of non-linear portfolios, which a parametric method is hard to apply. Of course, the other methods that are mentioned in the paper but not empirically studied such as EVT, CVaR and Monte-Carlo Methods should be assessed under the same framework that we use as well as in different settings to extend the research.

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